Return to 2-point function:

$$\langle \Omega | T\{\varphi(x)\varphi(y)\} | \Omega \rangle = \lim_{t \to \infty(1-i\epsilon)} \frac{\langle 0 | T\{\varphi_I(x)\varphi_I(y) \exp[-i\int_{-t}^{t} dt'\, H_I(t')]\} | 0 \rangle}{\langle 0 | T\{\exp[-i\int_{-t}^{t} dt'\, H_I(t')]\} | 0 \rangle}$$

First focus on the numerator. Suppressing the integration limits, write it as

$$\langle 0 | T\{\varphi_I(x)\varphi_I(y) \exp[-i\int dt\, H_I(t)]\} | 0 \rangle$$

Expand the exponential to get

$$\langle 0 | T\{ \overbrace{\varphi_I(x)\varphi_I(y)}^{\propto \lambda^0} + \overbrace{\varphi_I(x)\varphi_I(y)[-i\int dt\, H_I(t)]}^{\propto \lambda^1}$$

$$+ \underbrace{\cdots}_{\left(\substack{\text{higher order} \\ \text{in } \lambda}\right)} \} | 0 \rangle$$

The term $\propto \lambda^0$ (ie. indep of $\lambda$) is

$$\langle 0 | T\{\varphi_I(x)\varphi_I(y)\} | 0 \rangle = D_F(x-y)$$

Represent this diagrammatically as


$= D_F(x-y)$

Our first Feynman diagram!

The term $\propto \lambda^1 = \lambda$ is

$$\langle 0 | T\{ \varphi_I(x)\, \varphi_I(y)\, (-i) \int dt\, H_I(t) \} | 0 \rangle$$

(now use $H_I(t) = \int d^3 z\, \frac{\lambda}{4!}\, \varphi_I^4(z)$ )

$$= \langle 0 | T\{ \varphi_I(x)\, \varphi_I(y) \left(\frac{-i\lambda}{4!}\right) \int d^4 z\, \varphi_I(z)\varphi_I(z)\varphi_I(z)\varphi_I(z) \} | 0 \rangle$$

(here $\int d^4 z = \int d^3 z \int dt$ )

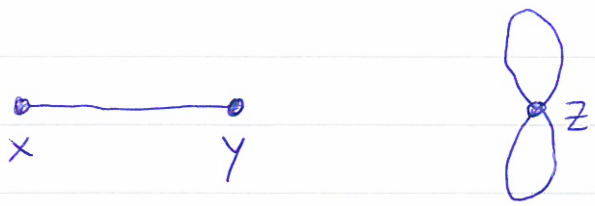
$$= \left(\frac{-i\lambda}{4!}\right) \int d^4 z\, \langle 0 | T\{ \varphi_I(x)\varphi_I(y)\, \varphi_I(z)\, \varphi_I(z)\, \varphi_I(z) \varphi_I(z)\} | 0 \rangle$$

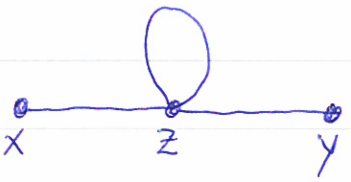
(now use Wick's theorem: 6 factors $\Rightarrow$ 5!! = 15 terms)
$\qquad\qquad\qquad\qquad\qquad\qquad$ $\Rightarrow$ 3 pairs

$$= \underset{\uparrow}{\overset{=(4-1)!!}{3}} \cdot \left(\frac{-i\lambda}{4!}\right) \underbrace{D_F(x-y)}_{\text{i.e. contracted } x \,\&\, y} \int d^4 z\, D_F(z-z)\, D_F(z-z)$$

$$+ \underset{\substack{\uparrow \\ 4\cdot 3 \cdot 1}}{12} \left(\frac{-i\lambda}{4!}\right) \int d^4 z\, D_F(x-z)\, D_F(y-z)\, D_F(z-z)$$

(Again, represent each of these expression as a Feynman diagram.)

$=$ 


$+$ 


amplitude for a free particle to propagate betw. a and b
↓
again, $D_F(a-b) =$ 


if $a = b$ : $D_F(a-a) =$ 


$x, y$ : <u>external</u> points

$z$ : an <u>internal</u> point

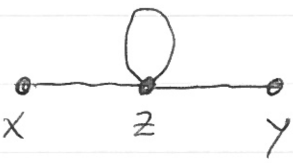
an internal point $z$ is called a <u>vertex</u>

- is associated with a factor $(-i\lambda \int d^4z)$
- <u>four</u> lines meet at a vertex

↑ b/c $\varphi^4$ theory

if a diagram is $\propto \lambda^n$ it has $n$ vertices (internal points)


The numerical factor is $\frac{3}{4!} = \frac{\cancel{3}}{1\cdot 2\cdot \cancel{3}\cdot 4} = \frac{1}{8}$




numerical factor : $\frac{12}{4!} = \frac{\cancel{3\cdot 4}}{1\cdot 2\cdot \cancel{3\cdot 4}} = \frac{1}{2}$

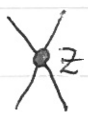
The numbers 8 and 2 here are known as the <u>symmetry factors</u> of the respective diagrams. (There exists alternative ways to calculate the symmetry factor based on the appearance of the diagram)

Numerator of 2-point function expression :

$$\boxed{\begin{aligned}&\langle 0 | T\{ \varphi_I(x)\varphi_I(y) \exp[-i\int dt\, H_I(t)]\} | 0\rangle \\ &= \text{the sum of all diagrams with two external points } x \text{ \& } y\end{aligned}}$$

(Position-space) Feynman rules for associating a mathematical expression to a Feynman diagram :

* For each propagator,  $= D_F(x-y)$

* For each vertex,  $= (-i\lambda)\int d^4z$

* Divide by the symmetry factor of the diagram

Calculating up to & including $O(\lambda)$ we have found that

$$\langle 0 | T \{ \varphi_I(x) \varphi_I(y) \exp[-i \int dt\, H_I(t)] \} | 0 \rangle$$

$=$ 

$O(\lambda^0)$

$+$ 

$O(\lambda^1)$

$+$ 

$O(\lambda^1)$

The last diagram contains a piece  that is <u>disconnected</u> from the external points x and y. We call such a diagram disconnected. The part of a diagram that is connected to the external point is called a connected piece. A diagram without any disconnected pieces is called connected.
So the first two diagram above are connected, while the third is disconnected.

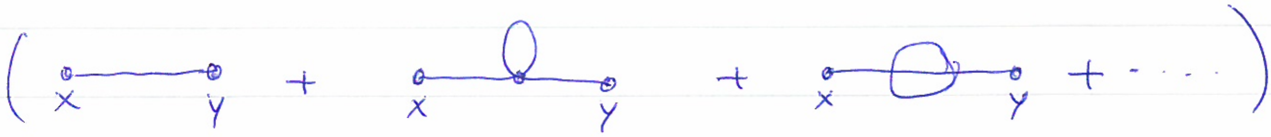
It can be shown that the numerator of

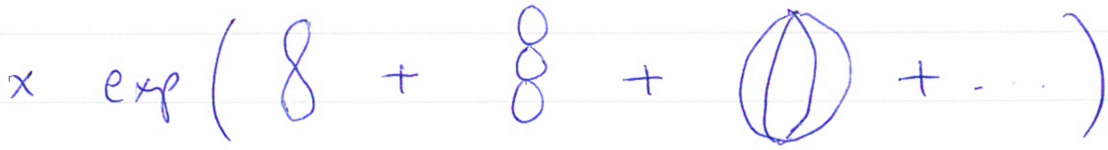
$$\langle 0 | T\{\varphi_I(x)\varphi_I(y)\exp[-i\int dt\, H_I(t)]\}|0\rangle$$

$$= (\text{sum of all } \underline{\text{connected}} \text{ diagrams})$$

$$\times \exp(\text{sum of all } \underline{\text{disconnected}} \text{ pieces})$$

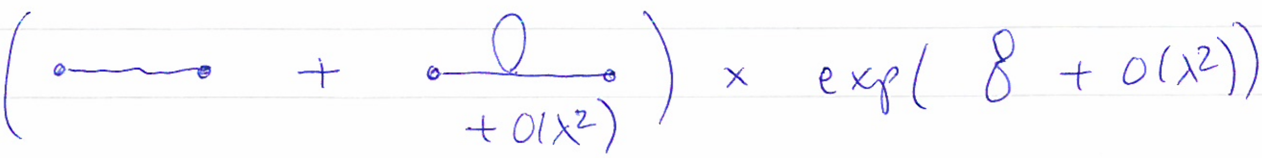
Pictorially, the rhs can be written

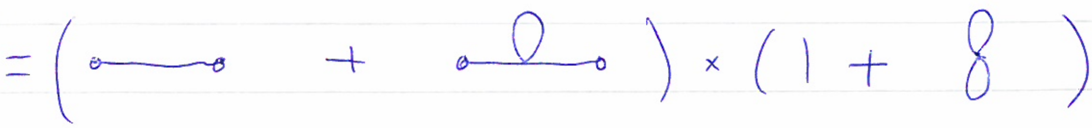



Let us verify that to $O(\lambda)$ this expression reduces to our result:

$O(\lambda^n)$: $n = \#$ of vertices



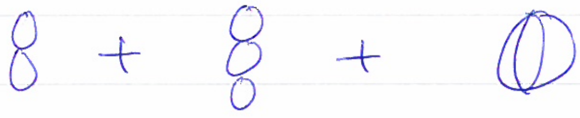


agrees with what we found

Furthermore, the denominator in $\langle \Omega | T\{\varphi(x)\varphi(y)\} | \Omega \rangle$ can be written (we won't show this either)

$$\langle 0 | T\{ \exp[-i \int dt\, H(t)] \} | 0 \rangle$$

$= \exp($ sum of all disconnected pieces $)$

$= \exp($  $+ \ldots)$

Thus the $\exp(\ldots)$ cancel between the numerator and denominator, so that

$$\langle \Omega | T\{ \varphi(x)\varphi(y) \} | \Omega \rangle$$

= sum of all connected diagrams

=  + . .