TFY4210, Quantum theory of many-particle systems, 2013: Tutorial 1

1. Fermionic creation and annihilation operators.

(a) Calculate $c_3 c_2^\dagger |1_1, 0_2, 1_3, \ldots\rangle$.

(b) Use the definitions of the operators $c_k^\dagger$ and c_k to show that these operators satisfy the defining property of adjoint operators, i.e.

$$\langle \bar{n}|c_k|n\rangle = \langle n|c_k^\dagger|\bar{n}\rangle^*. \tag{1}$$

Here $|n\rangle \equiv |n_1, n_2, \ldots\rangle$ and $|\bar{n}\rangle \equiv |\bar{n}_1, \bar{n}_2, \ldots\rangle$ are two arbitrary basis states for a fermionic many-particle system.

(c) Show that the fermionic creation and annihilation operators satisfy

$$\{c_k, c_l^\dagger\} = \delta_{k,l}. \tag{2}$$

In your proof, treat the two cases $k = l$ and $k \neq l$ separately (for the latter case you may limit your analysis to $k < l$).

(d) Use the fermionic anti-commutation relations to show that the fermionic number operator $\hat{n}_k \equiv c_k^\dagger c_k$ satisfies

$$\hat{n}_k^2 = \hat{n}_k. \tag{3}$$

Use this result to deduce the possible eigenvalues of $\hat{n}_k$.

2. Some useful commutator expressions.

(a) Show that, for arbitrary operators A, B, and C,

$$[AB, C] = A[B, C]_\zeta - \zeta[A, C]_\zeta B, \tag{4}$$

where $[A, B]_\zeta \equiv AB + \zeta BA$, with $\zeta = \mp 1$ corresponding to the commutator and anti-commutator, respectively.

(b) Show that, for both bosonic and fermionic creation and annihilation operators,

$$\begin{aligned} [\hat{n}_\alpha, \hat{a}_\beta^\dagger] &= \delta_{\alpha\beta}\, \hat{a}_\alpha^\dagger. && (5)\\ [\hat{n}_\alpha, \hat{a}_\beta] &= -\delta_{\alpha\beta}\, \hat{a}_\alpha. && (6) \end{aligned}$$