TFY4210, Quantum theory of many-particle systems, 2015: Tutorial 10

1. Matsubara and retarded Green functions for noninteracting bosons.

(a) Show that for noninteracting bosons with Hamiltonian

$$H_0 = \sum_\nu \xi_\nu c_\nu^\dagger c_\nu, \tag{1}$$

the Matsubara single-particle Green function $\mathcal{G}^{(0)}(\nu,\tau) = -\langle T_\tau(c_\nu(\tau)c_\nu^\dagger(0))\rangle$ is given by

$$\mathcal{G}^{(0)}(\nu,\tau) = -[\theta(\tau)(1+n_B(\xi_\nu)) + \theta(-\tau)n_B(\xi_\nu)]e^{-\xi_\nu\tau}. \tag{2}$$

[To prove this result, use that for noninteracting bosons, $\langle c_\nu^\dagger c_\nu\rangle = n_B(\xi_\nu)$, where

$$n_B(\xi_\nu) \equiv \frac{1}{e^{\beta\xi_\nu}-1} \tag{3}$$

is the Bose-Einstein distribution function.]

(b) Show that the Fourier transformed function is given by

$$\mathcal{G}^{(0)}(\nu, i\omega_n) = \frac{1}{i\omega_n - \xi_\nu}. \tag{4}$$

Use this to find the retarded Green function for noninteracting bosons.

2. Impurity scattering: Impurity average and Feynman diagrams.

In this problem j_1, j_2 etc. are defined as in Sec. 4.4 of the lecture notes, i.e. j_1 refers to the site summation variable in the first factor $\rho(\boldsymbol{k}-\boldsymbol{k}_1)$ in the product of ρ functions, j_2 refers to the site summation variable in the second factor $\rho(\boldsymbol{k}_1-\boldsymbol{k}_2)$, etc.

(a) Calculate the contribution to $\overline{\rho(\boldsymbol{k}-\boldsymbol{k}_1)\rho(\boldsymbol{k}_1-\boldsymbol{k}_2)\rho(\boldsymbol{k}_2-\boldsymbol{k}')}$ from the case $j_1 = j_2 \neq j_3$. Write down the corresponding contribution to $\overline{\mathcal{G}^{(3)}(\boldsymbol{k},\boldsymbol{k}')}$ and draw the corresponding Feynman diagram.

(b) Calculate the contribution to $\overline{\rho(\boldsymbol{k}-\boldsymbol{k}_1)\rho(\boldsymbol{k}_1-\boldsymbol{k}_2)\rho(\boldsymbol{k}_2-\boldsymbol{k}_3)\rho(\boldsymbol{k}_3-\boldsymbol{k}')}$ from the case $j_1 = j_3 \neq j_2 = j_4$. Write down the corresponding contribution to $\overline{\mathcal{G}^{(4)}(\boldsymbol{k},\boldsymbol{k}')}$ and draw the corresponding Feynman diagram.

3. Impurity scattering: Feynman diagrams at order $n = 4$.

(a) Draw all the Feynman diagrams that appear at order $n = 4$ in the perturbation expansion for the impurity-averaged single-particle imaginary-time (Matsubara) Green function. Group the diagrams according to the number of impurity crosses they contain.

(b) Identify (and draw) the new self-energy diagrams that appear at this order. Give the mathematical expression for each of them.