

TFY4210, Quantum theory of many-particle systems, 2013:

Tutorial 11

1. Matsubara and retarded Green functions for noninteracting bosons.

(a) Show that for noninteracting bosons with Hamiltonian

$$H_0 = \sum_{\nu} \xi_{\nu} c_{\nu}^{\dagger} c_{\nu}, \quad (1)$$

the Matsubara single-particle Green function $\mathcal{G}^{(0)}(\nu, \tau) = -\langle T_{\tau}(c_{\nu}(\tau)c_{\nu}^{\dagger}(0)) \rangle$ is given by

$$\mathcal{G}^{(0)}(\nu, \tau) = -[\theta(\tau)(1 + n_B(\xi_{\nu})) + \theta(-\tau)n_B(\xi_{\nu})]e^{-\xi_{\nu}\tau}. \quad (2)$$

[To prove this result, use that for noninteracting bosons, $\langle c_{\nu}^{\dagger} c_{\nu} \rangle = n_B(\xi_{\nu})$, where

$$n_B(\xi_{\nu}) \equiv \frac{1}{e^{\beta\xi_{\nu}} - 1} \quad (3)$$

is the Bose-Einstein distribution function.]

(b) Show that the Fourier transformed function is given by

$$\mathcal{G}^{(0)}(\nu, i\omega_n) = \frac{1}{i\omega_n - \xi_{\nu}}. \quad (4)$$

Use this to find the retarded Green function for noninteracting bosons.

2. Impurity scattering: Impurity average and Feynman diagrams.

In this problem j_1, j_2 etc. are defined as in Sec. 4.4 of the lecture notes, i.e. j_1 refers to the site summation variable in the first factor $\rho(\mathbf{k} - \mathbf{k}_1)$ in the product of ρ functions, j_2 refers to the site summation variable in the second factor $\rho(\mathbf{k}_1 - \mathbf{k}_2)$, etc.

(a) Calculate the contribution to $\overline{\rho(\mathbf{k} - \mathbf{k}_1)\rho(\mathbf{k}_1 - \mathbf{k}_2)\rho(\mathbf{k}_2 - \mathbf{k}')}$ from the case $j_1 = j_2 \neq j_3$. Write down the corresponding contribution to $\mathcal{G}^{(3)}(\mathbf{k}, \mathbf{k}')$ and draw the corresponding Feynman diagram.

(b) Calculate the contribution to $\overline{\rho(\mathbf{k} - \mathbf{k}_1)\rho(\mathbf{k}_1 - \mathbf{k}_2)\rho(\mathbf{k}_2 - \mathbf{k}_3)\rho(\mathbf{k}_3 - \mathbf{k}')}$ from the case $j_1 = j_3 \neq j_2 = j_4$. Write down the corresponding contribution to $\mathcal{G}^{(4)}(\mathbf{k}, \mathbf{k}')$ and draw the corresponding Feynman diagram.

3. Impurity scattering: Feynman diagrams at order $n = 4$.

(a) Draw all the Feynman diagrams that appear at order $n = 4$ in the perturbation expansion for the impurity-averaged single-particle imaginary-time (Matsubara) Green function. Group the diagrams according to the number of impurity crosses they contain.

(b) Identify (and draw) the new self-energy diagrams that appear at this order. Give the mathematical expression for each of them.