TFY4240, Electromagnetic theory, spring 2017:
Tutorial 1

Note that the Einstein summation convention is used throughout.

Problem 1.

Starting from the expression for the gradient ∇f in the cartesian coordinate system,

$$\nabla f = \frac{\partial f}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}, \tag{1}$$

show that this can be rewritten as the expression for ∇f in the cylindrical coordinate system,

$$\nabla f = \frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}. \tag{2}$$

Problem 2.

(a) Consider the identity

$$\epsilon_{ijk}\epsilon_{i\ell m} = \delta_{j\ell}\delta_{km} - \delta_{jm}\delta_{k\ell}. \tag{3}$$

For each of the following cases, explicitly write down and evaluate both the left hand side (lhs) and the right hand side (rhs) of the identity, and verify that they are equal:

1. $j = \ell = 1$ and $k = m = 3$
2. $j = m = 1$ and $k = \ell = 2$
3. $j = \ell = 1$, $k = 2$, $m = 3$

(b) Evaluate the following expressions:

1. δ_{ii}
2. $\delta_{ij}\epsilon_{ijk}$
3. $\epsilon_{ijk}\epsilon_{\ell jk}$
4. $\epsilon_{ijk}\epsilon_{ijk}$

Problem 3.

Show the following vector identities by making use of the Levi-Civita symbol:

$$\begin{array}{rl}
\text{a)} & \boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C}) = \boldsymbol{B}(\boldsymbol{A}\cdot\boldsymbol{C}) - \boldsymbol{C}(\boldsymbol{A}\cdot\boldsymbol{B}) \\
\text{b)} & \boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\times\boldsymbol{A}) = 0 \\
\text{c)} & \boldsymbol{\nabla}\times(\boldsymbol{\nabla}\psi) = 0 \\
\text{d)} & \boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{A}) = \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{A}) - \nabla^2\boldsymbol{A} \\
\text{e)} & \boldsymbol{\nabla}\times(f\boldsymbol{A}) = f(\boldsymbol{\nabla}\times\boldsymbol{A}) - \boldsymbol{A}\times(\boldsymbol{\nabla}f)
\end{array}$$