TFY4240

Solution problem set 2 Autumn 2015



Problem 1.

See Griffiths and lecture notes

Problem 2.

a) We want to find the potential V in some volume region Ω of a physical system, given (i) the charge density in Ω and (ii) the potential on the boundary of Ω . The method consists in constructing an alternative system where (i) and (ii) are the same as in the original one, and then solving the problem for the alternative system instead (because a uniqueness theorem guarantees that the solution for V in Ω will be the same in both systems). The alternative system contains "image" charges in the region outside of Ω , hence the name "method of images".

In this solution, we denote the Cartesian coordinates by (x, y, z) instead of (x_1, x_2, x_3) .

As the conductor is grounded, the potential has to be zero at all points in the conducting half-space, including at the surface z = -a:

$$V|_{z=-a} = 0.$$

The potential is continuous, so this is also the value of V immediately outside the conductor.

In addition, the normal derivative of the potential is discontinuous at the surface (due to the discontinuity of the electric field \mathbf{E} there):

$$-\varepsilon_0 \partial_z V|_{z=-a} = \sigma$$

Here σ is the surface charge density, and the derivative is evaluated immediately outside the conductor $(z = -a^+)$.

b) The region Ω of interest is here $z \ge -a$. This problem is solved in Griffiths, Chapter 3.2.1. After translating the coordinate system, the solution reads:

$$V(\mathbf{r}) = \frac{q}{4\pi\varepsilon_0} \left\{ \left[x^2 + y^2 + z^2 \right]^{-1/2} - \left[x^2 + y^2 + (z+2a)^2 \right]^{-1/2} \right\}.$$

Inserting z = -a, we easily see that the two terms cancel, and that V(z = -a) = 0.

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c) The region Ω of interest is here $-a \leq z \leq a$. This problem can seem tricky, but it is not so hard if you are systematic. We simply need to add up the contribution from an infinite number of point charges. The 'zeroth" term stems from the original point charge at the origin. Then, the next term stems from two 'mirror charges;" one for each plane. But, each of these mirror charges need a mirror charge for the opposite plane...and so on. Continuing this summation gives us positive and negative charges at:

$$q \text{ at } z = 4na,$$
 $n = 0, \pm 1, \pm 2, \dots$
 $-q \text{ at } z = (4n+2)a,$ $n = 0, \pm 1, \pm 2, \dots$

The analytical expression for V is somewhat involved and it was not a requirement to state this. It is simply found by summing up the contributions from all the point charges.

d) This is done by taking the normal derivative of the potential at the interface and using the discontinuity of the normal derivative of the field, $\partial_z V|_{z=-a}$. Griffiths does this for the 'simple" image problem in Chapter 3.2.1.