TFY4240, Electromagnetic theory, spring 2017: Tutorial 2

Problem 1.

(a) Write down an expression for the charge density $\rho(\boldsymbol{r}, t)$ for a system of N point charges q_α with positions $\boldsymbol{r}_\alpha(t)$ $(\alpha = 1, 2, \ldots, N)$.

(b) Find an expression for the current density $\boldsymbol{j}(\boldsymbol{r}, t)$. (Hint: Use the continuity equation.)

Problem 2.

Consider a system with a continuous charge distribution confined to the surface of a sphere with radius R. Taking the origin of a spherical coordinate system at the center of the sphere, the surface charge density is $\sigma(\theta, \phi)$. Find an expression for the (volume) charge density $\rho(r, \theta, \phi)$ at an arbitrary point in space. Show that this expression has the following necessary properties:

- it has the correct dimensions
- $\rho = 0$ for $r \neq R$
- it gives that the charge on an infinitesimal surface element with area da and angular coordinates (θ, ϕ) on the spherical surface is $\sigma(\theta, \phi)da$ (hint: do an appropriate integration over r from $r = R^-$ to $r = R^+$, where R^- and R^+ are infinitesimally smaller/bigger than R).

Problem 3.

The electric field due to a stationary charge density $\rho(\boldsymbol{r})$ is given by

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0} \int d^3r' \rho(\boldsymbol{r}') \frac{\hat{\boldsymbol{R}}}{R^2} \tag{1}$$

where $\boldsymbol{R} \equiv \boldsymbol{r} - \boldsymbol{r}'$. Show by direct calculation that $\nabla \times \boldsymbol{E} = 0$.

Problem 4.

Study the examples of Chapter 2 of Griffiths.

Problem 5.

(Adapted from the 2015 exam.)

A point charge q is held at a fixed position outside a grounded conducting sphere of radius R. The distance between the point charge and the center of the sphere is a. We take the z axis to pass through both the center of the sphere (where $z = 0$) and the point charge (where $z = a$). See Fig. 1 for an illustration of the geometry.

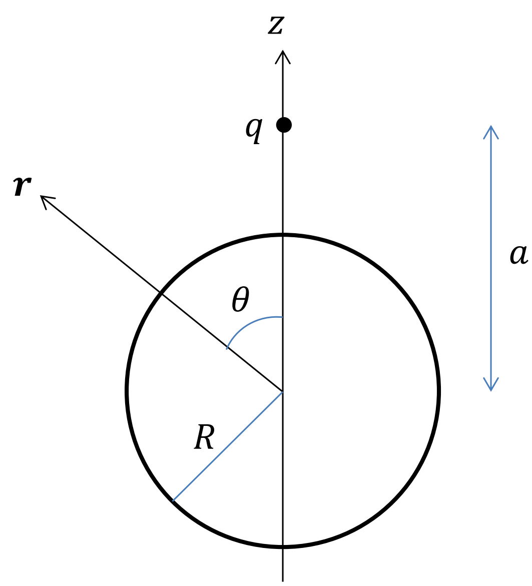

Figure 1: A point charge q outside a grounded conducting sphere of radius R.

(a) We wish to find the potential V at an arbitrary point $\boldsymbol{r}$ outside the sphere. Show that the problem can be solved by introducing an image charge q' where

$$q' = -\frac{R}{a}q, \tag{2}$$

which is positioned on the z axis at $z = b$ where

$$b = \frac{R^2}{a}. \tag{3}$$

Give an expression for the potential $V(\boldsymbol{r})$ outside the sphere.

(b) Find the induced surface charge density σ (which depends on the angle θ, see figure) and the total induced charge Q on the surface of the sphere.

(c) Suppose next that the conducting sphere is not grounded, but is instead held at a fixed potential $V_0 \neq 0$ (with respect to infinity, where $V \to 0$). Again we wish to find the potential everywhere outside the sphere. Solve this problem by introducing one more image charge. What is the charge and position of this second image charge?