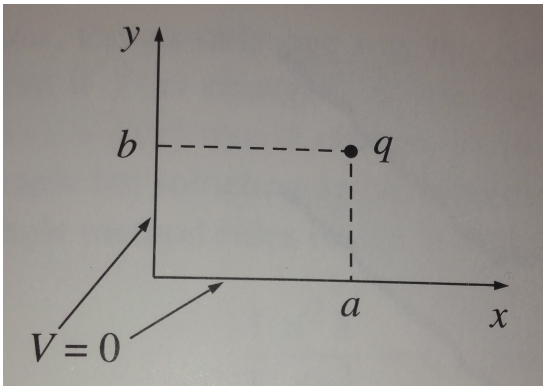
TFY4240, Electromagnetic theory, spring 2017:
Tutorial 3

Problem 1.



A grounded conductor has a surface consisting of two planes which meet at right angles (see the figure). The conductor extends indefinitely along the z axis. A point charge q is located outside the conductor at position $(x, y, z) = (a, b, 0)$.

(a) Find the potential V at an arbitrary point $(x > 0, y > 0, z)$ outside the conductor. [Hint: Use the method of images. You will need 3 image charges. Symmetry considerations will be useful to figure out their positions and charges.]

(b) Find the force on the charge q.

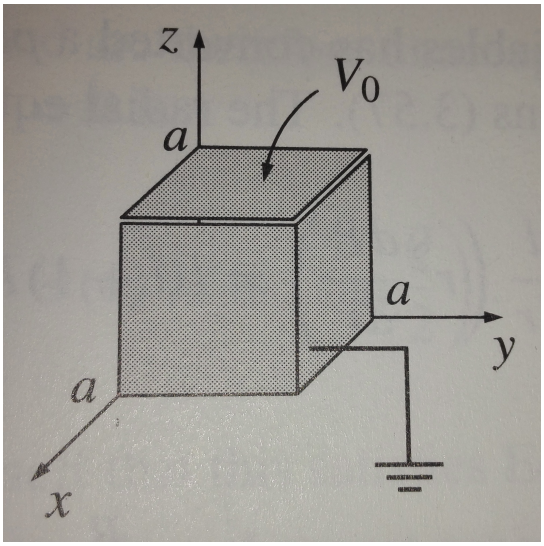
(This problem is a slightly rewritten version of part of Problem 3.10 in Griffiths, from which also the figure is taken.)

Problem 2.

Study examples 3.4 and 3.5 in Griffiths.

Problem 3.

A cubical box with sides of length a consists of five metal plates, which are welded together and grounded (see the figure). The top plate is made of a separate sheet of metal, insulated from the others, and held at a constant potential V_0. Find the potential inside the box. (This is Problem 3.15 in Griffiths, from which also the figure is taken).



Problem 4.

Let $f(x)$ be some function defined on the interval $[-1, 1]$.

Show that $f(x)$ can be expanded in terms of the Legendre polynomials $P_\ell(x)$ as

$$f(x) = \sum_{\ell=0}^{\infty} A_\ell P_\ell(x), \tag{1}$$

where

$$A_\ell = \frac{2\ell + 1}{2} \int_{-1}^{1} dx\, f(x) P_\ell(x). \tag{2}$$

[Hint: Use the orthogonality of the Legendre polynomials.]

(b) For the particular case that

$$f(x) = \begin{cases} -1 & \text{for } x < 0, \\ +1 & \text{for } x > 0, \end{cases} \tag{3}$$

argue without doing any calculation that the expansion (1) only will contain contributions from odd ℓ.

(c) For $f(x)$ given by Eq. (3), show that the first few terms of the expansion read

$$f(x) = \frac{3}{2} P_1(x) - \frac{7}{8} P_3(x) + \frac{11}{16} P_5(x) + \ldots. \tag{4}$$

[Here $P_1(x) = x$, $P_3(x) = \frac{1}{2}(5x^3 - 3x)$, and $P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$.]