NTNU

Institutt for fysikk

TFY4240

Problem set 4

Problem 1.
A "pure" (or "point") dipole at the origin gives the potential

$$V_{\rm dip}(\boldsymbol{r}) = \frac{\boldsymbol{p}\cdot\boldsymbol{r}}{4\pi\epsilon_0 r^3}. \tag{1}$$

Show that the electric field of the dipole can be written in the following "coordinate-free" form:

$$\boldsymbol{E}_{\rm dip}(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0 r^3}[3(\boldsymbol{p}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \boldsymbol{p}]. \tag{2}$$

Problem 2.
The electric potential on a spherical surface of radius R is given by

$$V(R,\theta) \;=\; V_0\cos^2\theta,$$

where V_0 is a constant. There is no charge outside or inside the spherical surface.

a) Show that the potential outside the spherical surface is given by

$$V_{\rm outside}(r,\theta) \;=\; \frac{V_0}{3}\left[\frac{R}{r} + 2\left(\frac{R}{r}\right)^3 P_2(\cos\theta)\right]. \tag{3}$$

b) Find the potential $V_{\rm inside}(r,\theta)$ inside the spherical surface.

c) Determine the surface charge density distribution on the spherical surface.

Problem 3.

a) A spherical surface of radius R has a surface charge density $\sigma(\theta)$, which can be expanded in Legendre polynomials as

$$\sigma(\theta) = \sum_{\ell=0}^{\infty}\sigma_\ell P_\ell(\cos\theta) \tag{4}$$

where $\{\sigma_\ell\}$ are the expansion coefficients. Show that this gives rise to a potential given by

$$V_{\rm outside}(r,\theta) \;=\; \frac{R}{\epsilon_0}\sum_{\ell=0}^{\infty}\frac{\sigma_\ell}{2\ell+1}\left(\frac{R}{r}\right)^{\ell+1} P_\ell(\cos\theta), \tag{5}$$

$$V_{\rm inside}(r,\theta) \;=\; \frac{R}{\epsilon_0}\sum_{\ell=0}^{\infty}\frac{\sigma_\ell}{2\ell+1}\left(\frac{r}{R}\right)^{\ell} P_\ell(\cos\theta). \tag{6}$$

b) Use the formulas in 3a to check your results for the potentials in 2a and 2b, given the surface charge density you found in 2c.

c) Use the formulas in 3a to find the potential produced by a dielectric sphere with uniform polarization $\boldsymbol{P}$. Then find the electric field outside and inside the sphere. [Hint: Choose the z direction to be the direction of $\boldsymbol{P}$. Find the volume bound charge density and the surface bound charge density.]

Problem 4.

Go through example 4.5 in Griffiths.