TFY4240

Problem set 5

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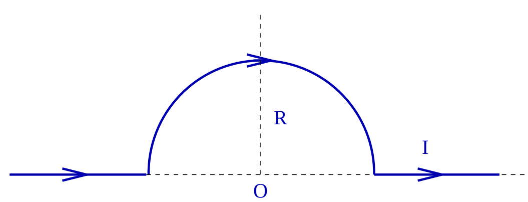
Problem 1.
[Overlaps with Ex. 4.7 in Griffiths] A sphere of a simple dielectric material with dielectric constant κ is placed in a uniform electric field $\boldsymbol{E}_0$.

a) Find the electric field inside and outside the sphere. [First find the potential, by making use of the formulation of the Poisson equation in terms of free charge, and the associated boundary conditions (including the boundary condition at infinity, which you need to determine; note that, given the presence of $\boldsymbol{E}_0$, this boundary condition is *not* $V \to 0$).]

b) Find the polarization $\boldsymbol{P}(\boldsymbol{r})$ and the total electric dipole moment $\boldsymbol{p}$ of the dielectric sphere.

Problem 2.
Go through Ex. 5.6 and 5.9 in Griffiths.

Problem 3.



An infinitely long wire carries a time-independent current I. The wire is bent so as to have a semi-circular detour, of radius R, around the origin O (see figure).

a) Derive an expression for the magnetic field $\mathbf{B}$ at the origin O of the coordinate system.

b) Determine the numeric value of this magnetic field given the current $I = 1$ A and radius $R = 1$ cm.

Problem 4.
[Problem 5.13 in Griffiths] A steady current I flows down an infinitely long cylindrical wire of radius a. Find the magnetic field, both inside and outside the wire, if

a) the current is uniformly distributed over the surface of the wire

b) the current is distributed in such a way that j is proportional to s, the distance from the axis

Problem 5.

Consider the expression for the static magnetic field due to a steady current distribution (the Biot-Savart law):

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int d^3r' \, \frac{\boldsymbol{j}(\boldsymbol{r}') \times \hat{\boldsymbol{R}}}{R^2} \tag{1}$$

where $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$. Show that this expression satisfies

a) $\nabla \cdot \boldsymbol{B}(\boldsymbol{r}) = 0$,

b) $\nabla \times \boldsymbol{B}(\boldsymbol{r}) = \mu_0 \boldsymbol{j}(\boldsymbol{r})$.

[For hints/help, see Sec. 5.3.2 in Griffiths.]