## TFY4240 Problem set 7 Autumn 2015



Problem 1.



A conducting rod can slide without friction along two parallel rails (see the figure). The distance between the rails is  $\ell$ . The rails are connected at one end by a resistance R, so that the system forms a closed circuit. The circuit is located in the horizontal plane, *i.e.* the xz-plane, with the rails oriented along the x-axis with their left edges at x = 0. The system is placed in a magnetic field B that lies in the xy-plane and forms an angle of 45° with the plane of the circuit. Hence it follows that  $B_x > 0$ ,  $B_y > 0$ , and  $B_z = 0$ , and we assume that the field strength is  $|\mathbf{B}| = B$ . The rod moves at constant velocity v towards the right.

- a) Calculate the magnetic flux through the circuit as function of the position x of the metal rod. Obtain an expression for the current in the circuit and determine its direction.
- b) Due to the current in the circuit and the external magnetic field, a force will act on the rod. Give the magnitude and direction of this force. What is the mechanical power needed to move the rod? Compare this power to the Ohmic heat loss in the resistance R.
- c) The force under point b) will also have a vertical component. For a sufficiently strong magnetic field B, the rod will leave the rail for a short period of time. Calculate the smallest value of B needed for this to happen when it is given that mass of the rod is m and g is the acceleration due to gravity.

We will next study how a magnetic field can be used to make an object levitate (Norwegian: "sveve"). This phenomenon is known as *magnetic levitation* and is used, for instance, in magnetic levitation train like the Maglev train in Shanghai.

Consider a conducting plane placed in the xz-plane, in which a uniform time-independent surface current density K is flowing in the positive z direction. For ease of calculation, we assume that the conducting plane is infinite in extent.

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- d) Determine the magnetic field B (magnitude and direction) in the region above the conducting plane. Show that the field is uniform in this region.
- e) A conducting rod is now placed above the plane so that it is parallel to K. If |K| is such that the magnitude of the magnetic field is B = 2.0 T, what is the smallest current I that has to go through the rod in order to levitate 1,000 kg/m of the rod? What direction must the current in the rod have relative to K?

## Problem 2.

Let  $\boldsymbol{v}$  be a general vector and let  $\stackrel{\leftrightarrow}{T}$  be a general tensor of rank 2. In terms of the basis vectors  $\hat{\boldsymbol{x}}_i$  (i = 1, 2, 3), one can write

$$egin{array}{rcl} m{v} &=& v_i \hat{m{x}}_i, \ ec{T} &=& T_{ij} \hat{m{x}}_i \hat{m{x}}_j \end{array}$$

where  $\hat{\boldsymbol{x}}_i \hat{\boldsymbol{x}}_j \equiv \hat{\boldsymbol{x}}_i \otimes \hat{\boldsymbol{x}}_j$ .

- **a)** Determine the components of  $\nabla \cdot \overset{\leftrightarrow}{T}$ .
- **b)** In addition to the dot product  $(\cdot)$  between a vector and a rank-2 tensor, one can also define a cross product  $(\times)$  between a vector and a rank-2 tensor. Like the dot product, the cross product is noncommutative. Unlike the dot product, whose result is a vector, the cross product gives another rank-2 tensor. The cross products  $\stackrel{\leftrightarrow}{T} \times \boldsymbol{v}$  and  $\boldsymbol{v} \times \stackrel{\leftrightarrow}{T}$  can be evaluated term by term using the definitions

$$egin{array}{rll} (\hat{m{x}}_i \hat{m{x}}_j) imes \hat{m{x}}_k &\equiv& \hat{m{x}}_i (\hat{m{x}}_j imes \hat{m{x}}_k), \ \hat{m{x}}_k imes (\hat{m{x}}_i \hat{m{x}}_j) &\equiv& (\hat{m{x}}_k imes \hat{m{x}}_i) \hat{m{x}}_j. \end{array}$$

Use these to find expressions for the components of  $\overset{\leftrightarrow}{T} \times \boldsymbol{v}$  and  $\boldsymbol{v} \times \overset{\leftrightarrow}{T}$ . (The result  $\hat{\boldsymbol{x}}_i \times \hat{\boldsymbol{x}}_j = \varepsilon_{ijk} \hat{\boldsymbol{x}}_k$  may be useful.)