TFY4240

Problem set 8 Autumn 2015

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Problem 1.
Consider a parallel-plate capacitor with circular plates of radius a and plate separation $w \ll a$. The capacitor is being charged due to a constant current I flowing through wires connected to the capacitor.

a) Find the electric and magnetic fields in the gap between the plates, as functions of the distance s from the axis and the time t. (Assume that the charge on the plates is zero at $t = 0$.)

b) Find the energy density u_{EM} and the Poynting vector $\boldsymbol{S}$ in the gap.

c) Verify that the differential version of Poynting's theorem,

$$\frac{\partial}{\partial t}(u_{\text{mech}} + u_{\text{EM}}) = -\nabla \cdot \boldsymbol{S}, \tag{1}$$

is satisfied, by explicitly calculating both sides of the equation.

d) Consider a cylindrical volume Ω inside the gap, whose axis goes through the plate centers, and which has an arbitrary radius $b < a$. Verify that the integral version of Poynting's theorem,

$$\frac{dW}{dt} + \frac{d}{dt}\int_{\Omega} u_{\text{EM}} d\tau = -\oint_{A} \boldsymbol{S} \cdot d\boldsymbol{a} \tag{2}$$

(where $\partial\Omega$ is the surface bounding Ω) is satisfied, again by explicitly calculating both sides of the equation.

Problem 2.
Problem 8.4 in Griffiths.

Problem 3.
Problem 9.11 in Griffiths.

Problem 4.
Consider a monochromatic plane wave that is traveling in the z direction and is linearly polarized in the x direction (i.e. $\boldsymbol{E}$ is parallel to the x axis).

a) Find all elements of the Maxwell stress tensor.

b) Verify that the differential version of the momentum conservation law (the momentum analogue of (1)) is satisfied, by explicitly calculating both sides of the equation.

c) Consider a box-shaped volume Ω with lengths L_x, L_y, and L_z. Verify that the integral version of the momentum conservation law (the momentum analogue of (2)) pertaining to Ω is satisfied, again by explicitly calculating both sides of the equation.