Problem 1

(a) $\vec{E}$ and $\vec{B}$ fields of the plane wave:

$$\vec{E}(z,t) = E_0 \cos(kz - \omega t + \delta)\,\hat{x}$$

$$\vec{B}(z,t) = \frac{1}{c} E_0 \cos(kz - \omega t + \delta)\,\hat{y}$$

Maxwell's stress tensor is defined as

$$T_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2}\delta_{ij} E^2\right) + \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij} B^2\right)$$

For the plane wave only E_x and B_y are nonzero

$\Rightarrow$ all nondiagonal terms of $\overleftrightarrow{T}$ are zero

For the diagonal terms we need

$$\epsilon_0 E^2 = \epsilon_0 E_x^2 = \epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)$$

$$\frac{1}{\mu_0} B^2 = \frac{1}{\mu_0} B_y^2 = \frac{1}{\mu_0}\frac{1}{c^2} E_0^2 \cos^2(kz - \omega t + \delta) = \epsilon_0 E^2$$

$$\Rightarrow T_{xx} = \epsilon_0 \left(\underbrace{E_x^2}_{=E^2} - \frac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(\underbrace{B_x^2}_{=0} - \frac{1}{2}B^2\right)$$

$$= \frac{\epsilon_0}{2}E^2 - \frac{\epsilon_0}{2}E^2 = \underline{\underline{0}}$$

$$T_{yy} = \epsilon_0 \left(\underbrace{E_y^2}_{=0} - \frac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(\underbrace{B_y^2}_{=B^2} - \frac{1}{2}B^2\right)$$

$$= -\frac{\epsilon_0}{2}E^2 + \frac{\epsilon_0}{2}E^2 = \underline{\underline{0}}$$

$$T_{zz} = \epsilon_0 \left(\underbrace{E_z^2}_{=0} - \frac{1}{2} E^2 \right) + \frac{1}{\mu_0} \left(\underbrace{B_z^2}_{=0} - \frac{1}{2} B^2 \right)$$

$$= -\frac{\epsilon_0}{2} E^2 \cdot 2 = -\epsilon_0 E^2 = -\epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)$$

In conclusion, the only nonzero component of $\overleftrightarrow{T}$ is

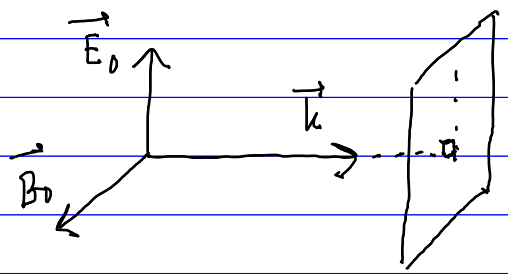
$$T_{zz} = -\epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)$$

$-\overleftrightarrow{T}$ is the momentum flux density, which means that $-T_{ij}$ is the momentum in the i direction crossing a surface oriented in the j direction, per unit area, per unit time

Thus for the plane wave,

$$-T_{zz} = \epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta) \qquad (= u_{EM})$$

is the z-momentum crossing a surface that is perpendicular to the plane wave, per unit area, per unit time:



(b) The differential form of the momentum conservation is in this case ($\vec{g}_{mech} = 0$ since there are no charges around)

$$\frac{\partial \vec{g}_{EM}}{\partial t} = \nabla \cdot \overleftrightarrow{T} \qquad \left(= -\nabla \cdot \left(-\overleftrightarrow{T} \right) \right)$$

Here $\vec{g}_{EM} = \dfrac{\vec{S}}{c^2}$. For the plane wave,

$$\vec{S} = c u_{EM} \hat{z} = c \epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta) \hat{z}$$

$$\Rightarrow \frac{\partial \vec{g}_{EM}}{\partial t} = \frac{1}{c^2} \frac{\partial \vec{S}}{\partial t} = \frac{1}{c} \hat{z} \frac{\partial u_{EM}}{\partial t}$$

$$= \hat{z} \frac{1}{c} \epsilon_0 E_0^2 \cdot 2\cos(kz - \omega t + \delta) \left(-\sin(kz - \omega t + \delta) \right) \cdot (-\omega)$$

$$= \hat{z} k \epsilon_0 E^2 \sin[2(kz - \omega t + \delta)]$$

Furthermore,

$$\nabla \cdot \overleftrightarrow{T} = \left(\hat{x} \partial_x + \hat{y} \partial_y + \hat{z} \partial_z \right) \cdot T_{zz} \hat{z}\hat{z} = \frac{\partial T_{zz}}{\partial z} \hat{z}$$

$$= -\hat{z} \epsilon_0 E_0^2 \frac{\partial}{\partial z} \cos^2(kz - \omega t + \delta)$$

$$= -\hat{z} \epsilon_0 E_0^2 \, 2\cos(kz - \omega t + \delta) \cdot \left(-\sin(kz - \omega t + \delta) \right) \cdot k$$

$$= \hat{z} k \epsilon_0 E_0^2 \sin[2(kz - \omega t + \delta)]$$

which is the same expression as for $\dfrac{\partial \vec{g}_{EM}}{\partial t}$. QED.

(c) In this case, since there are no charges around, $\vec{P}_{mech} = 0$, so the conservation law for momentum reads, in integral form,

$$\frac{d}{dt}\int_{\Omega} \vec{g}_{EM}\, d\tau = -\oint_{\partial\Omega} \left(-\overleftrightarrow{T}\right)\cdot d\vec{a}$$

Taking Ω to be a box with lengths L_x, L_y, L_z, the momentum stored in the EM field inside Ω is

$$\vec{P}_{EM} = \int_{\Omega} \vec{g}_{EM}\, d\tau = \underbrace{L_x L_y}_{\equiv A} \int_{z_0}^{z_0+L_z} dz \; \frac{\epsilon_0}{c} E_0^2 \cos^2(kz-\omega t+\delta)\,\hat{z}$$

$$= A\frac{\epsilon_0}{c}E_0^2\,\hat{z}\cdot \left.\frac{2(kz-\omega t+\delta)+\sin[2(kz-\omega t+\delta)]}{4k}\right|_{z_0}^{z_0+L_z}$$

$$= \hat{z}A\frac{\epsilon_0}{c}E_0^2\frac{1}{4k}\left\{2kL_z + \sin[2(k(z_0+L_z)-\omega t+\delta)] - \sin[2(kz_0-\omega t+\delta)]\right\}$$

The time rate of change of this momentum is

$$\frac{d}{dt}\vec{P}_{EM} = -2\omega A\hat{z}\frac{\epsilon_0}{c}E_0^2\frac{1}{4k}\left\{\cos[2(k(z_0+L_z)-\omega t+\delta)] - \cos[2(kz_0-\omega t+\delta)]\right\}$$

$$\overset{\omega=ck}{=} -\hat{z}\frac{A\epsilon_0 E_0^2}{2}\left\{\cos[2(k(z_0+L_z)-\omega t+\delta] - \cos[2(kz_0-\omega t+\delta)]\right\}$$

This should be equal to minus $\oint_{\partial\Omega}(-\overleftrightarrow{T})\cdot d\vec{a}$ where $\oint_{\partial\Omega}(-\overleftrightarrow{T})\cdot d\vec{a}$ is the momentum flowing out of Ω per unit time. Let us check this:

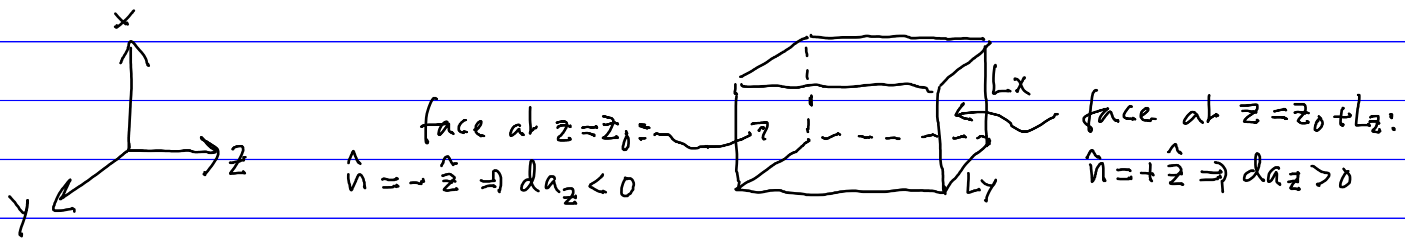
$$\overleftrightarrow{T} = T_{zz}\,\hat{z}\hat{z} \quad \text{(all other terms of } \overleftrightarrow{T} \text{ are zero)}$$

$$\Rightarrow \oint_{\partial\Omega} \overleftrightarrow{T} \cdot d\vec{a} = \oint_{\partial\Omega} T_{zz} \underbrace{\hat{z}\hat{z}\cdot(da_x \hat{x} + da_y \hat{y} + da_z \hat{z})}$$

$$= \hat{z}\,(da_x \underbrace{\hat{z}\cdot\hat{x}}_{=0} + da_y \underbrace{\hat{z}\cdot\hat{y}}_{=0} + da_z \underbrace{\hat{z}\cdot\hat{z}}_{=1})$$

$$= \hat{z} \oint_{\partial\Omega} T_{zz}\, da_z$$

The surface integral is a sum of contributions from the 6 faces of $\partial\Omega$. The 4 faces with $\hat{n} = \pm\hat{x}$ or $\hat{n} = \pm\hat{y}$ thus have $da_z = 0$ and therefore contribute zero. The face at $z = z_0$ has $\hat{n} = -\hat{z}$, so $da_z < 0 \Rightarrow da_z = -da$ while the face at $z = z_0 + L_z$ has $\hat{n} = +\hat{z}$, so $da_z > 0$ $\Rightarrow da_z = +da$, where da is a positive-valued area element.



The surface integral therefore becomes

$$\oint_{\partial\Omega} T_{zz}\, da_z = + \int_{z=z_0+L_z} T_{zz}\, da - \int_{z=z_0} T_{zz}\, da$$

Since T_{zz} does not depend on x or y, it is constant over each of the 2 faces and can be taken outside the integral. Using $\int da = L_x L_y = A$ then gives

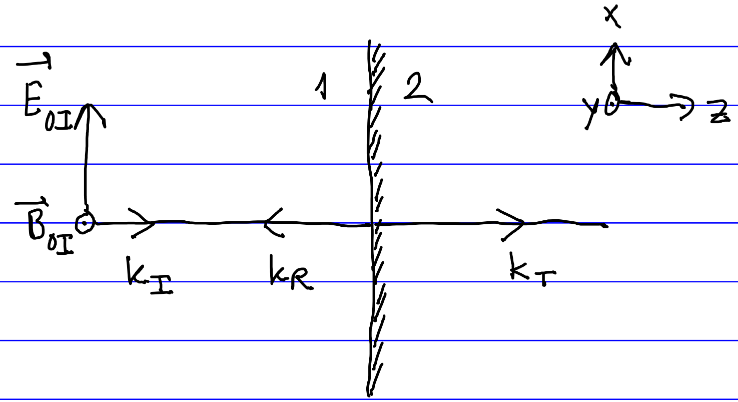
$$\oint_{\partial\Omega} T_{zz}\, da_z = A\left[T_{zz}(z=z_0+L_z) - T_{zz}(z=z_0)\right]$$

Inserting for T_{zz} gives $\oint_{\partial\Omega} \overleftrightarrow{T} \cdot d\vec{a}$

$$= -\hat{z} A \epsilon_0 E_0^2 \left[\cos^2(k(z_0+L_z) - \omega t + \delta) - \cos^2(kz_0 - \omega t + \delta)\right]$$

which equals the expression for $d\vec{p}_{EM}/dt$ (seen by using $\cos^2 x = \frac{1}{2}(\cos 2x + 1)$).

Problem 2



Normal incidence assumed, ie. $\theta_I = \theta_R = \theta_T = 0$

Boundary conditions (BC's):

(i) $\epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}$

(ii) $B_1^{\perp} = B_2^{\perp}$

(iii) $\vec{E}_1^{\parallel} = \vec{E}_2^{\parallel}$

(iv) $\vec{B}_1^{\parallel}/\mu_1 = \vec{B}_2^{\parallel}/\mu_2$

Since all $\vec{k}$-vectors ($\vec{k}_I$, $\vec{k}_R$, and $\vec{k}_T$) are $\perp$ to interface, and since both $\vec{E}$ and $\vec{B}$ are transverse (ie. $\vec{E} \perp \vec{k}$, $\vec{B} \perp \vec{k}$), we have $E_\perp = 0$ and $B_\perp = 0$, and therefore also $\vec{E}^{\parallel} = \vec{E}$ and $\vec{B}^{\parallel} = \vec{B}$ for all waves (incident, reflected and transmitted). Thus BC's (i) and (ii) are trivially satisfied, and BC's (iii) and (iv) become

(iii′) $\vec{E}_1 = \vec{E}_2$

(iv′) $\vec{B}_1/\mu_1 = \vec{B}_2/\mu_2$

To consider (iii′) we use $\vec{E}_1 = \vec{E}_I + \vec{E}_R$, $\vec{E}_2 = \vec{E}_T$, where

Incident $\vec{E}$: $\tilde{\vec{E}}_I(z,t) = \tilde{E}_{0I}\, e^{i(k_1 z - \omega t)}\, \hat{x}$

Reflected $\vec{E}$: $\tilde{\vec{E}}_R(z,t) = \tilde{E}_{0R}\, e^{i(-k_1 z - \omega t)}\, \hat{n}_R$

Transmitted $\vec{E}$: $\tilde{\vec{E}}_T(z,t) = \tilde{E}_{0T}\, e^{i(k_2 z - \omega t)}\, \hat{n}_T$

where $\hat{n}_R = \hat{x}\cos\varphi_R + \hat{y}\sin\varphi_R$
$\hat{n}_T = \hat{x}\cos\varphi_T + \hat{y}\sin\varphi_T$

At the boundary, $z=0 \Rightarrow e^{i(\cdots\cdot)} = e^{-i\omega t}$ is the same for all fields and can be cancelled. Thus (iii') gives

$$\tilde{E}_{0I}\,\hat{x} + \tilde{E}_{0R}\,\hat{n}_R = \tilde{E}_{0T}\,\hat{n}_T$$

$$\Rightarrow \tilde{E}_{0I}\,\hat{x} + \tilde{E}_{0R}\left(\cos\varphi_R\,\hat{x} + \sin\varphi_R\,\hat{y}\right) = \tilde{E}_{0T}\left(\cos\varphi_T\,\hat{x} + \sin\varphi_T\,\hat{y}\right)$$

Separately equating the x & y components of this vector equation gives

$$\tilde{E}_{0I} + \tilde{E}_{0R}\cos\varphi_R = \tilde{E}_{0T}\cos\varphi_T \qquad \text{(iii}'\text{-a)}$$
$$\tilde{E}_{0R}\sin\varphi_R = \tilde{E}_{0T}\sin\varphi_T \qquad \text{(iii}'\text{-b)}$$

To consider (iv') we use that $\vec{B}_1 = \vec{B}_I + \vec{B}_R$, $\vec{B}_2 = \vec{B}_T$, where

Incident $\vec{B}$: $\tilde{\vec{B}}_I(z,t) = \frac{1}{v_1}\hat{k}_I \times \tilde{\vec{E}}_I(z,t)$

$$= \frac{1}{v_1}\hat{z} \times \tilde{E}_{0I}\, e^{i(k_1 z - \omega t)}\,\hat{x}$$

$$= \frac{1}{v_1}\tilde{E}_{0I}\, e^{i(k_1 z - \omega t)}\,\hat{y}$$

Reflected $\vec{B}$: $\tilde{\vec{B}}_R(z,t) = \frac{1}{v_1}\hat{k}_R \times \tilde{\vec{E}}_R(z,t)$

$$= \frac{1}{v_1}(-\hat{z}) \times \tilde{E}_{0R}\, e^{i(-k_1 z - \omega t)}\left[\hat{x}\cos\varphi_R + \hat{y}\sin\varphi_R\right]$$

$$= \frac{1}{v_1}\tilde{E}_{0R}\, e^{i(-k_1 z - \omega t)}\left[-\hat{y}\cos\varphi_R + \hat{x}\sin\varphi_R\right]$$

Transmitted $\vec{B}$: $\vec{B}_T(z,t) = \frac{1}{v_2}\vec{k}_T \times \tilde{\vec{E}}_T(z,t)$

$$= \frac{1}{v_2}\hat{z} \times \tilde{E}_{0T}\, e^{i(k_2 z - \omega t)}\left[\hat{x}\cos\varphi_T + \hat{y}\sin\varphi_T\right]$$

$$= \frac{1}{\sqrt{2}} \tilde{E}_{0T} e^{i(k_2 z - \omega t)} \left[\hat{y} \cos\varphi_T - \hat{x} \sin\varphi_T \right]$$

At the boundary the $e^{i(\dots)}$ factors are again identical and thus cancel in (iv'), which gives

$$\frac{1}{\mu_1 v_1} \left[\hat{x} \tilde{E}_{0R} \sin\varphi_R + \hat{y} \left(\tilde{E}_{0I} - \tilde{E}_{0R} \cos\varphi_R \right) \right]$$

$$= \frac{1}{\mu_2 v_2} \left[-\hat{x} \tilde{E}_{0T} \sin\varphi_T + \hat{y} \tilde{E}_{0T} \cos\varphi_T \right]$$

which gives the following equations for the x and y components

$$\frac{1}{\mu_1 v_1} \tilde{E}_{0R} \sin\varphi_R = -\frac{1}{\mu_2 v_2} \tilde{E}_{0T} \sin\varphi_T \qquad \text{(iv'-a)}$$

$$\frac{1}{\mu_1 v_1} \left(\tilde{E}_{0I} - \tilde{E}_{0R} \cos\varphi_R \right) = \frac{1}{\mu_2 v_2} \tilde{E}_{0T} \cos\varphi_T \qquad \text{(iv'-b)}$$

If $\sin\varphi_R \neq 0$, (iii'-b) and (iv'-a) imply $\sin\varphi_T \neq 0$ as well. Assuming $\sin\varphi_R, \sin\varphi_T \neq 0$, divide (iii'-b) by (iv'-a). This gives $\mu_1 v_1 = -\mu_2 v_2$, which is an equation without physical solutions (at least for conventional materials). If we nevertheless accept it for now, insert it into (iii'-a) and (iv'-b), and divide the former by the latter, we get (with $Z \equiv (\tilde{E}_{0R}/\tilde{E}_{0I}) \cos\varphi_R$)

$$(1+Z)/(1-Z) = -1 \Rightarrow 1 + Z = Z - 1 \Rightarrow 1 = -1$$

which obviously isn't even mathematically possible. Thus we conclude from (iii'-b) and (iv'-a) that $\sin\varphi_R = \sin\varphi_T = 0$ $\Rightarrow \cos\varphi_R, \cos\varphi_T = \pm 1$, corresponding to angles 0 or π, which gives $\hat{n}_R, \hat{n}_T = \pm \hat{x}$. But the sign of $\hat{n}$ is not physical (see footnote 2 in Ch. 9 of Griffiths; another way of seeing this is that we have just 2 eqs left [(iii'-a) & (iv'-b)], so there cannot be more unknowns than $\tilde{E}_{0R}$ and $\tilde{E}_{0T}$. So we just pick the signs. The most convenient choice is $\hat{n}_R = \hat{n}_T = +\hat{x}$, i.e. $\varphi_R = \varphi_T = 0$.

Problem 3

We have

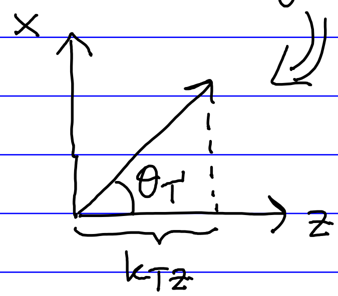
$$\tilde{\vec{E}}_T = \tilde{\vec{E}}_{0T}\, e^{i(\vec{k}_T\cdot\vec{r}-\omega t)}$$

where $\vec{k}_T = (k_{Tx}, k_{Ty}, k_{Tz})$

$$k_{Tx} = k_{Ix} \quad \text{(Griffiths Eq. (9.97))}$$
$$= k_I \sin\theta_I$$
$$= (\omega/v_1)\sin\theta_I = \frac{\omega}{c} n_1 \sin\theta_I \equiv k \quad \text{(real)}$$

$$k_{Ty} = k_{Iy} \quad \text{(Griffiths Eq. (9.96))}$$
$$= 0$$

$$k_{Tz} = k_T \cos\theta_T \quad \Leftarrow \text{ from the geometric def of } \theta_T$$



$$\Rightarrow k_{Tz}^2 = k_T^2 \cos^2\theta_T$$

$$= \left(\frac{\omega}{c} n_2\right)^2 \left(1 - \sin^2\theta_T\right)$$

$$= \left(\frac{\omega}{c}\right)^2 n_2^2 \left(1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_I\right)$$

$$= \left(\frac{\omega}{c}\right)^2 \left(n_2^2 - n_1^2 \sin^2\theta_I\right)$$

$$= \left(\frac{\omega}{c}\right)^2 n_1^2 \left[\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_I\right]$$

$$= \left(\frac{\omega}{c}\right)^2 n_1^2 \left[\sin^2\theta_c - \sin^2\theta_I\right]$$

If $\theta_I > \theta_c$, we see that $k_{Tz}^2 < 0$, so k_{Tz} is imaginary $\Rightarrow k_{Tz} = \pm i\kappa$ with

$$\kappa \equiv \frac{\omega}{c} n_1 \sqrt{\sin^2\theta_I - \sin^2\theta_c} \quad \text{(real and positive)}$$

Inserting this into $e^{i\vec{k}_T\cdot\vec{r}}$, the z-dependence becomes

$$e^{i(\pm i\kappa)z} = e^{\mp\kappa z}$$

The bottom sign implies exponential growth of the transmitted wave as it propagates further into medium 2. This does not make sense physically, so the top sign must be chosen, i.e. $k_{Tz} = +i\kappa$

Therefore

$$\tilde{\vec{E}}_T = \tilde{\vec{E}}_{0T}\, e^{-\kappa z}\, e^{i(kx-\omega t)} \qquad \text{(in medium 2, ie } z \geq 0\text{)}$$

(We note that $k_{Tz} = k_T \cos\theta_T = + i\kappa$ gives $\cos\theta_T = i\frac{\kappa}{k_T}$ which is a number on the positive imaginary axis, giving $\cos\theta_T = + i\sqrt{\sin^2\theta_T - 1}$ as stated in the problem text).

(b) We have, for p-polarization

$$\frac{\tilde{E}_{0R}}{\tilde{E}_{0I}} = \frac{\alpha-\beta}{\alpha+\beta} = \frac{\frac{\cos\theta_T}{\cos\theta_I}-\beta}{\frac{\cos\theta_T}{\cos\theta_I}+\beta} = \frac{\cos\theta_T - \beta\cos\theta_I}{\cos\theta_T + \beta\cos\theta_I}$$

$$= \frac{ic - \beta\cos\theta_I}{ic + \beta\cos\theta_I} \qquad \left(\text{where } c \equiv \frac{\kappa}{k_T} \text{ is real}\right)$$

$$= \frac{c + i\beta\cos\theta_I}{c - i\beta\cos\theta_I} \qquad (\text{note that } \beta\cos\theta_I \text{ is real too})$$

Define the complex number $w = c + i\beta\cos\theta_I \equiv \rho e^{i\phi}$ where $\rho = |w|$ and $\phi = \arg(w)$

$$\Rightarrow \frac{\tilde{E}_{0R}}{\tilde{E}_{0I}} = \frac{\rho e^{i\phi}}{\rho e^{-i\phi}} = e^{2i\phi}$$

, i.e. a complex number of magnitude 1 ("unit modulus")

The reflection coefficient R is

$$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \frac{|\tilde{E}_{0R}|^2}{|E_{0I}|^2} = \left|\frac{\tilde{E}_{0R}}{\tilde{E}_{0I}}\right|^2 \quad (*)$$

where E_{0R} and E_{0I} are the (real) amplitudes of the reflected and incident waves, and we used $E_{0R} = |\tilde{E}_{0R}|$ and $E_{0I} = |\tilde{E}_{0I}|$. (Note that $(*)$ corresponds to

$R = \left|\frac{\alpha-\beta}{\alpha+\beta}\right|^2$, which generalizes Eq. (9.115) in Griffiths

to situations with α complex.) For our case we thus get

$$\underline{\underline{R}} = |e^{2i\phi}|^2 = \underline{\underline{1}}$$

(c) The time-averaged Poynting vector for the transmitted wave is

$$\langle \vec{S}_T \rangle = \frac{1}{2\mu_2} \mathrm{Re}\left[\tilde{\vec{E}}_T^* \times \tilde{\vec{B}}_T\right]$$

Using $\tilde{\vec{E}}_T = \tilde{\vec{E}}_{0T}\, e^{i(\vec{k}_T\cdot\vec{r}-\omega t)}$ and Faraday's law,

$$\nabla \times \tilde{\vec{E}}_T = -\tilde{\vec{E}}_{0T} \times \nabla e^{i(\vec{k}_T\cdot\vec{r}-\omega t)} \qquad \text{(product rule 7)}$$

$$= i\vec{k}_T \times \tilde{\vec{E}}_T = -(-i\omega)\tilde{\vec{B}}_T$$

$$\Rightarrow \tilde{\vec{B}}_T = \frac{1}{\omega}\vec{k}_T \times \tilde{\vec{E}}_T$$

$$\Rightarrow \langle \vec{S}_T \rangle = \frac{1}{2\mu_2\omega} \mathrm{Re}\left[\vec{E}_T^* \times (\vec{k}_T \times \tilde{\vec{E}}_T)\right]$$

Using the triple product rule, this becomes

$$\langle \vec{S}_T \rangle = \frac{1}{2\mu_2 \omega} \mathrm{Re}\left[\vec{k}_T (\tilde{\vec{E}}_T \cdot \tilde{\vec{E}}_T) - \tilde{\vec{E}}_T (\tilde{\vec{E}}_T^* \cdot \vec{k}_T) \right]$$

where $\tilde{\vec{E}}_T^* \cdot \tilde{\vec{E}}_T = |\tilde{\vec{E}}_T|^2$ (real) and

$$\tilde{\vec{E}}_T^* \cdot \vec{k}_T = \tilde{E}_{Tx}^* k_{Tx} + \tilde{E}_{Tz}^* k_{Tz} \quad \text{(y-components are 0)}$$

$$= \tilde{E}_{Tx}^* k_{Tx} + \tilde{E}_{Tz}^* i\kappa$$

From $\nabla \cdot \tilde{\vec{E}}_T = 0$ follows $\vec{k}_T \cdot \tilde{\vec{E}}_T = 0$.
Taking the complex conjugate gives

$$0 = \vec{k}_T^* \cdot \tilde{\vec{E}}_T^* = \tilde{E}_{Tx}^* k_{Tx} - \tilde{E}_{Tz}^* i\kappa = 0$$

$$\Rightarrow \tilde{\vec{E}}_T^* \cdot \vec{k}_T = 2i\kappa \tilde{E}_{Tz}^*$$

$$\Rightarrow \langle \vec{S}_T \rangle = \frac{1}{2\mu_2 \omega} \mathrm{Re}\left[\vec{k}_T |\tilde{\vec{E}}_T|^2 - \tilde{\vec{E}}_T 2i\kappa \tilde{E}_{Tz}^* \right]$$

$$\Rightarrow \langle \vec{S}_T \rangle \cdot \hat{z} = \frac{1}{2\mu_2 \omega} \mathrm{Re}\left[k_{Tz} |\tilde{\vec{E}}_T|^2 - \tilde{E}_{Tz} 2i\kappa \tilde{E}_{Tz}^* \right]$$

$$= \frac{1}{2\mu_2 \omega} \mathrm{Re} \underbrace{\left[i\left(|\tilde{\vec{E}}_T|^2 - 2|\tilde{E}_{Tz}|^2 \right)\kappa \right]}_{\text{purely imaginary} \Rightarrow \mathrm{Re}[\cdot\cdot]=0} = \underline{\underline{0}}$$

The transmission coefficient is $T = \dfrac{I_T}{I_I}$

where $I_T = |\langle \vec{S}_T \rangle \cdot \hat{z}|$, $I_I = |\langle \vec{S}_I \rangle \cdot \hat{z}|$

Thus since $\langle \vec{S}_T \rangle \cdot \hat{z} = 0$, $I_T = 0 \Rightarrow \underline{\underline{T = 0}}$

We also see that $R + T = 1$, so energy is conserved, as expected.