Assignment 1, TFY4240 Electromagnetic Theory midterm project, Spring 2017

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In assignment 1, we will solve the Laplace equation in 2 dimensions by the method of separation of variables; this is a variation of Example 3.4 in Griffiths. The problem is to determine the potential in a long, square, hollow tube, where the four walls have different potentials. The boundary conditions are as follows:

$$
\begin{aligned}
V(x=0,y) &= 0, \\
V(x=L,y) &= 0, \\
V(x,y=0) &= 0, \\
\text{and } V(x,y=L) &= V_0(x).
\end{aligned}
\tag{1}
$$

Your assignment is as follows:

1. Set up the equation you need to solve, and make it dimensionless (e.g. use $\xi = x/L$ as a variable instead of x).

2. Formulate a solution in terms of Fourier coefficients, in a way suitable for numerical calculation.

3. Write a computer program that takes any function $V_0(x)$, and calculates $V(x,y)$ inside the square tube. It should be possible to choose to what order your Fourier series should be summed up to, in order to investigate convergence issues.

4. Plot $V_0(x)$, $V(x,y)$, and $V(x,y)$ on the boundaries (e.g. $V(x,y=L)$) to verify fulfillment of the boundary conditions.

5. Check how fast the calculation converges towards the correct result. In detail, you should compare how many Fourier orders you need for different V_0 functions.

6. Calculate and plot $\vec{E}(x,y)$ using your method of choice.

To plot the potential, $V(x, y)$, there are several possibilities. Possibilities include using a contour plot (`pyplot.contour`) or a filled contour plot (`pyplot.contourf`). To plot the electric field, there are two possible approaches. One is to simply plot the vector arrows (`pyplot.quiver`) with their length being proportional to the electric field. Another possibility is to plot the vector length $|\vec{E}(x, y)|$ as a contour plot, and separately (or on top) plot the field direction by using vector arrows of length 1 (unit vectors). Check out the Matplotlib gallery for inspiration: `http://matplotlib.sourceforge.net/gallery.html`

Some example functions you can play with are:

$$V_0(x) = \begin{cases} V_c \sin(\frac{m\pi x}{L}), & m = 1, 2, 3, \ldots \\ V_c \left[1 - \left(\frac{x}{L} - \frac{1}{2}\right)^4\right] \\ V_c \theta(x - L/2)\theta(3L/4 - x) \end{cases} \tag{2}$$

Here, V_c is a constant that you can choose freely[1], and θ is the Heaviside step function. Play around with some of your own functions as well, and have fun!

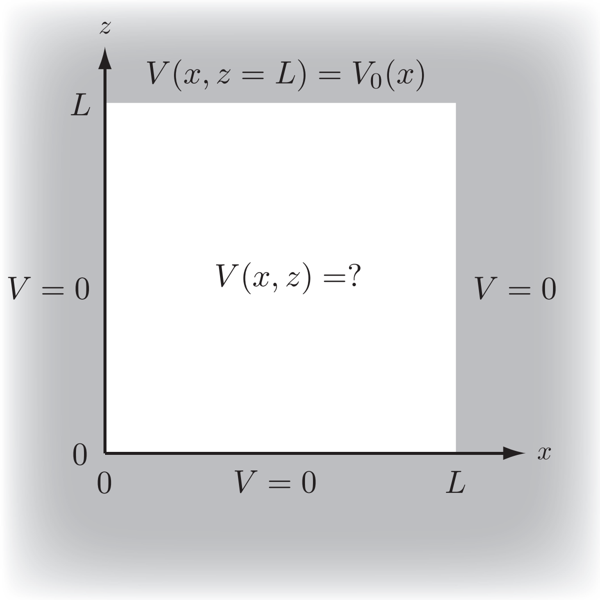

Figure 1: Sketch of the problem geometry.

[1]You should "eliminate" V_c in problem 1.