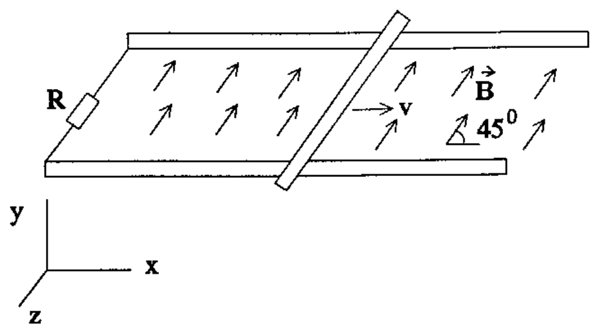
TFY4240
Problem set 8

NTNU

Institutt for fysikk

Problem 1.



A conducting rod can slide without friction along two parallel rails (see the figure). The distance between the rails is ℓ. The rails are connected at one end by a resistance R, so that the system forms a closed circuit. The circuit is located in the horizontal plane, *i.e.* the xz-plane, with the rails oriented along the x-axis with their left edges at $x = 0$. The system is placed in a magnetic field $\boldsymbol{B}$ that lies in the xy-plane and forms an angle of 45° with the plane of the circuit. Hence it follows that $B_x > 0$, $B_y > 0$, and $B_z = 0$, and we assume that the field strength is $|\boldsymbol{B}| = B$. The rod moves at constant velocity v towards the right.

a) Calculate the magnetic flux through the circuit as function of the position x of the metal rod. Obtain an expression for the current in the circuit and determine its direction.

b) Due to the current in the circuit and the external magnetic field, a force will act on the rod. Give the magnitude and direction of this force. What is the mechanical power needed to move the rod? Compare this power to the Ohmic heat loss in the resistance R.

c) The force under point b) will also have a vertical component. For a sufficiently strong magnetic field B, the rod will leave the rail for a short period of time. Calculate the smallest value of B needed for this to happen when it is given that mass of the rod is m and g is the acceleration due to gravity.

We will next study how a magnetic field can be used to make an object levitate (Norwegian: "sveve"). This phenomenon is known as *magnetic levitation* and is used, for instance, in magnetic levitation train like the Maglev train in Shanghai.

Consider a conducting plane placed in the xz-plane, in which a uniform time-independent surface current density $\boldsymbol{K}$ is flowing in the positive z direction. For ease of calculation, we assume that the conducting plane is infinite in extent.

d) Determine the magnetic field $\boldsymbol{B}$ (magnitude and direction) in the region above the conducting plane. Show that the field is uniform in this region. [Hint: See Ex. 5.8 in Griffiths. Alternatively, calculate the magnetic field by making use of the result for the field from an infinitely long current-carrying wire (the surface current configuration can be viewed as an infinite continuous set of parallel wires).]

e) A conducting rod is now placed above the plane so that it is parallel to $\boldsymbol{K}$. If $|\boldsymbol{K}|$ is such that the magnitude of the magnetic field is $B = 2.0$ T, what is the smallest current I that has to go through the rod in order to levitate $1,000$ kg/m of the rod? What direction must the current in the rod have relative to $\boldsymbol{K}$?

Problem 2.
Consider a parallel-plate capacitor with circular plates of radius a and plate separation $w \ll a$. The capacitor is being charged due to a constant current I flowing through wires connected to the capacitor.

a) Find the electric and magnetic fields in the gap between the plates, as functions of the distance s from the axis and the time t. (Assume that the charge on the plates is zero at $t = 0$.)

b) Find the energy density u_{EM} and the Poynting vector $\boldsymbol{S}$ in the gap.

c) Verify that the differential version of Poynting's theorem,

$$\frac{\partial}{\partial t}(u_{\mathrm{mech}} + u_{\mathrm{EM}}) = -\nabla \cdot \boldsymbol{S}, \tag{1}$$

is satisfied, by explicitly calculating both sides of the equation.

d) Consider a cylindrical volume Ω inside the gap, whose axis goes through the plate centers, and which has an arbitrary radius $b < a$. Verify that the integral version of Poynting's theorem,

$$\frac{dW}{dt} + \frac{d}{dt}\int_{\Omega} u_{\mathrm{EM}} d\tau = -\oint_{a} \boldsymbol{S} \cdot d\boldsymbol{a} \tag{2}$$

(where a is the closed surface bounding Ω) is satisfied, again by explicitly calculating both sides of the equation.

Problem 3.
Go through Example 8.2 (Maxwell stress tensor calculation of the force on one hemisphere of a uniformly charged solid sphere) and Example 8.4 (about conservation of angular momentum) in Griffiths.

Problem 4.
Problem 8.4 in Griffiths.

Problem 5.
Problem 9.11 in Griffiths.